Appendix A. The surfaces

We will follow the scheme below:
G: the Galois group.
G^0: the index 2 subgroup of the elements that do not exchanges the factors.
In the follow $\mathfrak{S}_n$ will denote the symmetric group in n letters, $D_{p,q,r}$ the generalized dihedral group with presentation: $D_{p,q,r} = \langle x, y | x^p, y^q, xyx^{-1}y^{-r} \rangle$ and $D_n := D_{2,n,-1}$ is the usual dihedral group of order $2n$.
$G(a,b)$ denotes the b-th group of order a in the MAGMA database of groups.
T: the type of the system of spherical generators.
L: here we list a set of elements of G that is a spherical generators system for G^0 that gives the surface.
H_1: the first homology group of the surface.
π_1: the fundamental group of the surface.

A.1. $K^2 = 1$, basket $\{2 \times A_1 + 2 \times A_3\}$.

A.1.1. *Galois group.* $(\mathbb{Z}_2)^3 \rtimes_\varphi \mathbb{Z}_4 : \varphi(1) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$G := \langle (2,5,6,8)(3,7),\ (1,2)(3,5)(4,6)(7,8),\ (1,3)(2,5)(4,7)(6,8),$
$(2,6)(5,8),\ (1,4)(2,6)(3,7)(5,8) \rangle < \mathfrak{S}_8$
G^0: $D_4 \times \mathbb{Z}_2$
T: $(2,2,2,4)$
L: $(1,8)(2,7)(3,6)(4,5)$, $(1,7)(2,8)(3,4)(5,6)$, $(1,3)(2,8)(4,7)(5,6)$, $(1,5,4,8)(2,7,6,3)$
H_1: $\mathbb{Z}_4$
π_1: $\mathbb{Z}_4$

A.2. $K^2 = 2$, basket $\{6 \times A_1\}$.

A.2.1. *Galois group.* $(\mathbb{Z}_2)^2 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G:=$\langle (1,2,4,6)(3,5,7,8),\ (2,5)(6,8),\ (1,3)(2,5)(4,7)(6,8),$
$(1,4)(2,6)(3,7)(5,8) \rangle < \mathfrak{S}_8$
G^0: $(\mathbb{Z}_2)^3$
T: $(2,2,2,2,2)$
L: $(1,3)(4,7)$, $(1,7)(2,6)(3,4)(5,8)$, $(1,3)(2,5)(4,7)(6,8)$, $(2,5)(6,8)$, $(1,7)(2,6)(3,4)(5,8)$
H_1: $\mathbb{Z}_2 \times \mathbb{Z}_4$
π_1: $\mathbb{Z}_2 \times \mathbb{Z}_4$

A.2.2. *Galois group:* Sylow 2-subgroup of the Suzuki group $Sz(8)$
G: $\langle x_1, x_2, x_3, y \mid x_i^4, y^2, [x_i, y], x_3^2 y, [x_1, x_2]y, [x_3, x_2]x^2 y, [x_2, x_3]x_1^2 x_2^2 \rangle$
G^0:$\langle z_1, z_2, w \mid z_i^4, w^2, [z_i, w], [z_1, z_2]w \rangle$,
it can be view as $(\mathbb{Z}_2 \times \mathbb{Z}_4) \rtimes_\varphi \mathbb{Z}_4$ where $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$
T:$(4,4,4)$
L: x_1^3, $x_1 x_2 x_1^2$, $x_1 x_2 x_1 y x_2^2$
H_1: $(\mathbb{Z}_2)^3$

π_1: $(\mathbb{Z}_2)^3$

A.3. $K^2 = 2$, basket $\{A_1 + 2 \times A_3\}$.

A.3.1. *Galois group:* $(\mathbb{Z}_2)^4 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

G: $\langle(2,6,7,12)(3,9,10,16)(4,11)(8,14,15,13),$
$(1,2)(3,6)(4,7)(5,8)(9,13)(10,14)(11,15)(12,16),$
$(1,3)(2,6)(4,9)(5,10)(7,13)(8,14)(11,16)(12,15),$
$(2,7)(3,10)(6,12)(8,15)(9,16)(13,14),$
$(1,4)(2,7)(3,9)(5,11)(6,13)(8,15)(10,16)(12,14),$
$(1,5)(2,8)(3,10)(4,11)(6,14)(7,15)(9,16)(12,13)\rangle < \mathfrak{S}_{16}$
G^0: $(\mathbb{Z}_2)^4 \rtimes_\psi \mathbb{Z}_2$, $\psi(1) = \varphi(2)$
T: $(2,2,2,4)$
L: $(2,7)(3,10)(6,12)(8,15)(9,16)(13,14),$
$(1,16)(2,12)(3,11)(4,10)(5,9)(6,15)(7,14)(8,13),$
$(1,14)(2,10)(3,8)(4,12)(5,6)(7,16)(9,15)(11,13),$
$(1,2,4,7)(3,14,9,12)(5,8,11,15)(6,16,13,10)$
H_1: $\mathbb{Z}_4$
π_1: $\mathbb{Z}_4$

A.3.2. *Galois group:* $(\mathbb{Z}_3)^2 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$

G: $\langle(1,2)(3,4,5,6),\ (3,5)(4,6),\ (2,4,6),\ (1,3,5)(2,4,6)\rangle < \mathfrak{S}_6$
G^0: $(\mathbb{Z}_3)^2 \rtimes_\psi \mathbb{Z}_2$, $\psi(1) = \varphi(2)$
T: $(2,2,3,3)$
L: $(3,5)(4,6),\ (2,6)(3,5),\ (1,3,5),\ (1,5,3)(2,4,6)$
H_1: $\mathbb{Z}_3$
π_1: $\mathbb{Z}_3$

A.4. $K^2 = 4$, basket $\{4 \times A_1\}$.

A.4.1. *Galois group:* $D_{2,8,5} \rtimes_\varphi \mathbb{Z}_2$, $\varphi(1) = \begin{cases} x \mapsto x \\ y \mapsto yxy^4 \end{cases}$

G: $\langle(1,2,3,6,4,5,7,8),\ (2,5)(3,7),\ (2,5)(6,8),\ (1,3,4,7)(2,6,5,8),$
$(1,4)(2,5)(3,7)(6,8)\rangle < \mathfrak{S}_8$
G^0: $D_4 \times \mathbb{Z}_2$
T: $(2,2,2,2,2)$
L: $(2,5)(6,8), (1,7)(2,6)(3,4)(5,8), (1,4)(2,5), (1,4)(2,5), (1,7)(2,8)(3,4)(5,6)$
H_1: $\mathbb{Z}_2 \times \mathbb{Z}_8$
π_1: $(\mathbb{Z}_2)^2 \rtimes_\psi \mathbb{Z}_8$, $\psi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

A.4.2. *Galois group:* $((\mathbb{Z}_2)^2 \rtimes_\varphi \mathbb{Z}_4) \times \mathbb{Z}_2$, $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G: $\langle(1,2,5,8)(3,7,10,14)(4,6,11,13)(9,12,15,16),\ (2,6)(7,12)(8,13)(14,16),$
$(1,3)(2,7)(4,9)(5,10)(6,12)(8,14)(11,15)(13,16),$

$(1,4)(2,6)(3,9)(5,11)(7,12)(8,13)(10,15)(14,16),$
$(1,5)(2,8)(3,10)(4,11)(6,13)(7,14)(9,15)(12,16)\rangle < \mathfrak{S}_{16}$
G^0: $\mathbb{Z}_2^4$
T: $(2,2,2,2,2)$
L: $(1,5)(2,13)(3,10)(4,11)(6,8)(7,16)(9,15)(12,14),$
$(1,3)(2,12)(4,9)(5,10)(6,7)(8,16)(11,15)(13,14),$
$(1,4)(3,9)(5,11)(10,15),$
$(1,10)(2,16)(3,5)(4,15)(6,14)(7,13)(8,12)(9,11),$
$(1,4)(2,6)(3,9)(5,11)(7,12)(8,13)(10,15)(14,16)$
H_1: $(\mathbb{Z}_2)^3 \times \mathbb{Z}_4$
π_1: $\langle p_1,\, p_2,\, p_3,\, p_4 \mid$
$p_1^2,\, p_3^2, (p_3p_2)^2,\, (p_1, p_2^{-1})^2,\, p_4p_2^{-1}p_4^{-1}p_2^{-1},\, p_4p_1p_3p_4^{-1}p_3p_1,\, (p_1p_4^2)^2,\, (p_4^{-2}p_3)^2\rangle$

A.4.3. *Galois group:* Sylow 2-subgroup of a double cover of the Suzuki group $Sz(8)$
G: $\langle x_1,\, x_2,\, x_3 \mid x_1^4, x_2^4, x_3^4,\, [x_1^{-1}, x_2^{-1}]x_3^2,\, x_3x_1x_2^{-1}x_3^{-1}x_1x_2,\, x_3^{-1}x_2x_3x_1^2x_2,$
$x_3^2[x_1^{-1}, x_2],\, x_3x_2^{-1}x_3x_1^{-1}x_3^{-1}x_2x_3x_1,\, x_3x_1^{-1}x_2^2x_3x_1x_3x_2^2x_3^{-1}x_2^2\rangle$
G^0: $\langle z_1,\, z_2 \mid z_1^4,\, z_2^4,\, [z_1, z_2^2],\, (z_1^{-1}z_2z_1z_1)^2,\, (z_2z_1^{-1})^4,\, (z_1^2z_2)^4,\, [z_1^2, z_2]^2\rangle,$
T: $(4,4,4)$
L: $(x_3x_2)^2x_1^2x_3^3,\, x_2^2x_3^2,\, (x_2x_3)^2x_2x_1^2x_3^3$
H_1: $(\mathbb{Z}_2)^3$
π_1: $(\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes_\psi \mathbb{Z}_2,\, \psi(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$

A.5. $K^2 = 8$**, basket** $\emptyset$**.**

A.5.1. *Galois group:* $(D_{2,8,5} \rtimes_\varphi \mathbb{Z}_2) \times \mathbb{Z}_2,\, \varphi(1) = \begin{cases} x \mapsto x \\ y \mapsto yxy^4 \end{cases}$
G: $\langle(1,2,4,8,5,9,12,16)(3,7,10,15,11,6,13,14),$
$(2,6)(4,12)(7,9)(8,15)(10,13)(14,16),$
$(1,3)(2,7)(4,10)(5,11)(6,9)(8,15)(12,13)(14,16),$
$(1,3)(2,6)(4,10)(5,11)(7,9)(8,14)(12,13)(15,16),$
$(1,4,5,12)(2,8,9,16)(3,10,11,13)(6,14,7,15),$
$(1,5)(2,9)(3,11)(4,12)(6,7)(8,16)(10,13)(14,15)\rangle < \mathfrak{S}_{16}$
G^0: $D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
T: $(2,2,2,2,2)$
L: $(1,5)(2,7)(3,11)(6,9)(8,14)(15,16),$
$(2,7)(4,12)(6,9)(8,14)(10,13)(15,16),$
$(1,13)(2,8)(3,12)(4,11)(5,10)(6,14)(7,15)(9,16),$
$(1,4)(2,14)(3,10)(5,12)(6,8)(7,16)(9,15)(11,13),$
$(1,3)(2,6)(4,10)(5,11)(7,9)(8,14)(12,13)(15,16)$
H_1: $(\mathbb{Z}_2)^3 \times \mathbb{Z}_8$
π_1: $1 \to \Pi_{17} \times \Pi_{17} \to \pi_1 \to G \to 1$

A.5.2. *Galois group:* $G(256, 3678)$
G: $\langle x_1,\, x_2,\, x_3 \mid x_1^4,\, x_2^4,\, x_3^4,\, [x_1^{-1}, x_2^{-1}]x_3^2,\, x_2x_3x_1^2x_2x_3^{-1},\, x_1x_3x_2x_1x_3^{-1}x_2^{-1},$
$x_1x_2x_3x_2^{-1}x_1x_3,\, x_2^2x_3x_1^{-1}x_3x_1,\, x_1(x_3x_2)^{-2}x_1x_3^2,\, x_2^{-1}x_1x_2x_1^2x_3^2x_1,$
$x_2^2(x_1x_3^{-1})^2x_1^2,\, x_3^2x_1^{-1}x_3^2x_2x_3^2x_1x_2^{-1}\rangle$
G^0: $G(128, 36)$,

$\langle z_1,\, z_2 \mid z_1^4,\, z_2^4,\, [z_1^2, z_2^2],\, (z_2^{-1}z_1z_2z_1)^2,\, (z_2^{-1}z_1)^4,\, (z_2z_1^{-1}z_2z_1)^2,$
$[z_1^{-1}, z_2^{-1}]^2,\, [z_2z_1^2z_2^{-1}, z_1],\, [z_2z_1z_2, z_1^2],\, [z_2^2, z_1^{-1}]^2\rangle,$
T: $(4, 4, 4)$
L: $x_2x_3,\, x_3x_2^{-1}x_3^{-1}x_2x_3x_1^{-1}x_3x_1x_3x_2x_3^{-2}x_2x_3^2,\, x_2^{-1}x_3^2x_2^{-1}x_3^{-1}x_2x_3$
H_1: $(\mathbb{Z}_4)^3$
π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

A.5.3. *Galois group:* $G(256, 3678)$
G: as above
G^0: $G(128, 36)$, as above
T: $(4, 4, 4)$
L: $x_1x_3^{-2}x_1^{-1}x_3x_1x_3x_2^2,\, x_3x_2^{-2}x_3^2x_2^{-1}x_3^{-1}x_2x_3,\, x_1x_3^{-1}x_2^{-2}x_3^2x_1^{-1}x_3x_1x_3x_2x_3^{-2}x_2x_3^2$
H_1:$(\mathbb{Z}_2)^4 \times \mathbb{Z}_4$
π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

A.5.4. *Galois group:* $G(256, 3678)$
G: as above
G^0: $G(128, 36)$, as above
T: $(4, 4, 4)$
L: $x_1x_3x_2^{-2}x_3^2x_2^{-1}x_3^{-2}x_2x_3^2,\, x_1x_2x_3x_1^{-1}x_3x_1x_3x_2^2,\, x_2x_3^{-2}x_1^{-1}x_3x_1x_3x_2^2$
H_1: $(\mathbb{Z}_2)^2 \times (\mathbb{Z}_4)^2$
π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

A.5.5. *Galois group:* $G(256, 3679)$
G: $\langle x_1,\, x_2,\, x_3 \mid x_1^4,\, x_2^4,\, x_3^4,\, [x_1^{-1}, x_2^{-1}]x_3^2,\, x_2x_1^{-1}x_3x_2x_3x_1^{-1},\, x_2x_3x_2x_1^2x_3^{-1},$
$x_1x_3x_2x_1^{-1}x_2x_3^{-1},\, x_2x_3x_2x_1^2x_3^{-1},\, x_3x_2x_1x_3x_1x_2^{-1},\, [x_3, x_1]x_1^{-1}x_2x_1x_2,$
$[x_3^{-1}, x_2]x_1^2x_2^2,\, (x_3^{-1}x_2)^4,\, (x_2^{-1}x_1)^2x_3^{-1}x_1x_3x_1\rangle$
G^0: $G(128, 36)$, as above
T: $(4, 4, 4)$
L: $x_2x_3,\, x_3x_2^{-1}x_3^{-1}x_2x_3x_1x_3^2x_1^{-1}x_3^{-2}x_2^{-1}x_3^{-2}x_2x_3^{-2},\, x_2^{-1}x_3^2x_2^{-1}x_3^{-1}x_2x_3$
H_1: $(\mathbb{Z}_2)^2 \times (\mathbb{Z}_4)^2$
π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$